Using Control Theory to Model the Long-term Economic Effects of Wildfire[1]

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Abstract

Wildland fire management strategies often have long-term economic and ecological impacts, as evidenced by the increase in fire danger resulting from the total suppression policy of the last several decades. In the long run, the choice of an optimal wildland fire management strategy depends upon the cumulative effects of fire management factors as well as the interaction between them. A theoretical extension to the cost plus net value change (C+NVC) model is developed by using the principles of control theory. It explores the long-term relationships among the factors of production and the choice of optimal management strategies given that fire management actions have consequences in the future.

Introduction

The cost plus net value change (C+NVC) framework has been the most widely used economic fire management tool since its inception in 1916 (Headly). Although the model has evolved to account for changing management philosophy (Gorte and Gorte 1979, Pyne 1996), further adaptations will enable managers to meet the objectives of ecosystem management. Wildfire managers are broadening their focus from individual fires and annual budgeting concerns to include ecosystem-wide objectives and long-term effects. Such objectives include long-term cost efficiency, sustainability of fire programming, and the consideration of ecological effects (Williams and others 1993). These objectives are important aspects of ecosystem management and fire planning, yet are not adequately addressed by existing models.

The C+NVC model minimizes the sum of fire management expenditures plus the net change in resource value for damaging wildfires. Total costs include annual expenditures on suppression and presuppression. Presuppression, or program level, is a combination of fire management activities (prevention, detection, and fuels management) that constitute the fire management mix (Mills and Bratten 1988). Once the program level has been defined, the optimal combination of presuppression activities is then determined (Gonázlez-Cabán and others 1986).

Research to improve various components of the C+NVC model has included efforts to reduce program cost or to improve the efficiency of the fire management mix derived from the least cost program level. For example, Bellinger and others (1983) analyzed the cost effectiveness of resulting program levels and determined that the program cost was appropriate, yet efficiency could be improved by reallocating management mix activities. Similarly, González-Cabán and others (1986) used the C+NVC framework to demonstrate that an efficient management mix can be determined, given the optimum program level. Mills and Bratten (1982) developed the Forest Economics Evaluation System (FEES) to address cost effectiveness and efficiency of C+NVC based programs. Upon testing it (1988) they determined that the total program cost was almost solely a function of presuppression. Finally, Hesseln and others (1998) developed a theoretical extension to the C+NVC using catastrophe theory. They modeled the production function for wildfire behavior and subsequently related environmental and ecological effects to economic outcomes of wildland fire management (Hesseln and others 1999). These research efforts, however, do not consider the long run.

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Fire management activities often have profound long-term effects on the ecology of a region, and over time, affect how the landscape responds to wildfire (Wade and Lundsford 1990, Weber and Taylor 1992). Similarly, the effects of current fire management investments, such as fuels reduction, will likely be manifested in the future through reduced hazard of catastrophic fire and resultant ecological, physical, and financial damage. The C+NVC model, as it is currently used, does not embody the theoretical association between fire management programs and ecological effects elicited by those programs, and therefore, does not address long-term ecosystem management objectives. Furthermore, theoretical extensions of the model to address long-term sustainability of fire management programs-combinations of presuppression and suppression-have not been developed, thereby ignoring potentially important economic ramifications of various fire management activities.

To better enable managers to address long-term economic objectives, we develop a theoretical extension to the C+NVC model by using control theory. Our objectives are to explore the long-term economic relationships among fire management activities and physical and financial damage, and to investigate the applicability of control theory to formulate a long-term optimization model. Our analysis is an extension of the C+NVC cusp model (Hesseln and others 1998, Hesseln and others 1999) that embodies environmental and ecological effects of fire behavior. We begin first with a review of the C+NVC model and then discuss the principles of control theory in the context of fire management modeling. Finally, we conclude with a discussion of long-term fire management planning.

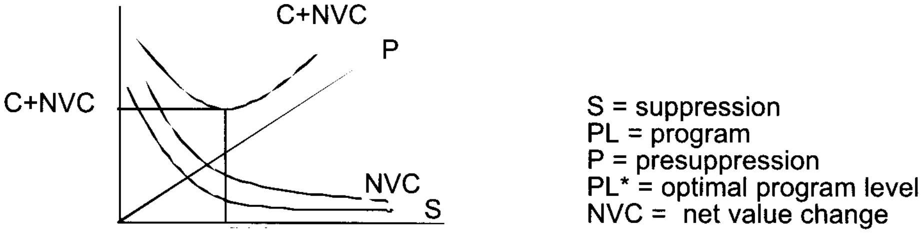
Evolution of the C+NVC Model

The C+NVC model was developed to minimize the sum of fire management expenditures plus the net change in resource value resulting from wildfire. The relationship between fire management expenditures and net value change is specified by equation [1]:

$$C + NVC = W^S S + W^P P + NVC(S,P) \qquad [1]$$

in which cost C is the sum of suppression and presuppression expenditures S and P evaluated at their prices W^S and W^P. The net change in resource value NVC is a function of management activities S and P *(fig. 1)*.

Figure 1
Cost plus net value change (NVC).



Presuppression expenditures in dollars, is represented along the x-axis *(fig. 1)*. It is assumed presuppression is inversely related to suppression and net value change; thus, as presuppression expenditures increase, suppression expenditures and NVC decrease. When the curves are added vertically, the resulting bowl-shaped curve represents the total cost plus net value change *(fig. 1)*. The optimal level of presuppression corresponds with the minimum of the C+NVC curve. The optimal level of presuppression is also known as program level or preparedness and represents the annual programming budget for the USDA Forest Service. For example, the National Fire Management Analysis System (NFMAS) uses C+NVC as an economic basis to estimate the expected annual cost of fire management (Brandel 1988) where the resulting program level is calculated by using historical program levels and net value change figures. However, past fire management programs are not directly associated with future programming needs or ecological effects in subsequent years (USDA 1993).

Although the model has undergone several changes in response to changing management philosophy, it fails to incorporate significant factors. First, the model is not based on fire behavior, which is often erratic and seemingly unpredictable. Second, there is no consideration of environmental factors that can widely influence fire behavior and subsequent, physical, ecological and financial damage. Furthermore, although the relationships among suppression, presuppression, and net value change are somewhat intuitive, such relationships have never been tested. Finally, the model is static and does not consider the long-term relationships between fire management factors and ecological outcomes.

In an effort to address these problems, Hesseln and others (1998) modeled a production function for fireline intensity by using a cubic equation. Equation [2] represents fireline intensity I as a function of a and b which are linear combinations of ecological and environmental factors, windspeed, initial 1-hour fuel moisture, and fuel loading, expressed by equations [3] and [4]:

$$V'(I) = I^3 + bI + a = 0 \qquad [2]$$

$$a = a_0 + a_1(windspeed) + a_2(fuel\text{-}loading) + a_3(fuel\text{-}moisture) \qquad [3]$$

$$b = b_0 + b_1(windspeed) + b_2(fuel\text{-}loading) + b_3(fuel\text{-}moisture) \qquad [4]$$

Physical and financial damage are then directly related to fireline intensity through equation [5]:

$$NVC = f(I) \qquad [5]$$

which is substituted into the C+NVC equation [1] to produce equation [6]:

$$C+NVC=W^sS+W^PP+NVC(I(S,P),A(S,P),R) \qquad [6]$$

Cost plus net value change is thus dependent upon fire behavior through fireline intensity (I), which is a function of environmental and ecological variables (a, b). This expression expands the range of expected C+NVC values, given the volatility of fire behavior and environmental factors. To solve for the optimal levels of suppression and presuppression, we differentiate equation [6] with respect to S and P.

$$\frac{\partial(C+NVC)}{\partial S} = W^S + \frac{\partial NVC}{\partial I}\cdot\frac{\partial I}{\partial S} + \frac{\partial NVC}{\partial A}\cdot\frac{\partial A}{\partial S} = 0 \qquad [7]$$

$$\frac{\partial(C+NVC)}{\partial P} = W^P + \frac{\partial NVC}{\partial I}\cdot\frac{\partial I}{\partial P} + \frac{\partial NVC}{\partial A}\cdot\frac{\partial A}{\partial P} = 0 \qquad [8]$$

Equations [7] and [8] state that suppression and presuppression will be optimal where the marginal cost of a fire management activity defined by its price is equal to the marginal benefit resulting from such activity. Furthermore, equations [7] and [8] indicate the marginal effectiveness of S and P on the reduction in damage through both fire control via fireline intensity and containment via area burned (Hesseln and others 1998).

Although the cubic model is based on fire management behavior and environmental parameters, it does not evaluate long-term effectiveness of fire management activities. The C+NVC model expressed by equation [6] does not reflect the cumulative nature of management actions and ecosystem response over time. Cost in one period is currently directly related to management expenditure in that same period and resulting net value change or expected damage. Furthermore, it ignores the complex relationships among suppression, presuppression, and net value change. To effectively evaluate fire management programs, the model could be specified over the long term to capture the investment return relationships between fire management activities and long-term effects. Control theory may provide a method by which to evaluate long-term fire management activities.

Principles of Control Theory

Control theory is used to optimize problems where decisions are related through time. Rather than optimizing a variable in a single time period, we recognize that decisions are dynamic in that the choice of a decision variable in one time period will affect future choices of that decision variable. Similarly, the decision variable will also affect ecological and economic outcomes throughout the planning period, further complicating the choice of optimal management variables. Therefore, to optimize decision making, we seek to determine the optimal time path of decision variables over a specified period (Silberberg 1990). There is ample evidence that fire management is dynamic and could be enhanced if considered in this context.

The relationship between fire management activities, particularly prescribed fire, and net value change has become evident as the effects of past fire management programs manifest themselves in a changing ecosystem (Arno and Brown 1991). For example, a past policy of complete fire exclusion without prescribed burning led to ecosystem changes resulting in increased costs and losses from a higher incidence of fire and disease. In ecosystems where the natural fire frequency is relatively high, the detrimental effects of fire exclusion are just now being realized (Mutch 1994). The total suppression policy intended to eliminate fire and reduce damage has, in some ecosystems, exchanged present damage for future damage. Omi and Kalabokidis (1991) studied the effects of fire on intensively managed lands by comparing forests in Yellowstone National Park with adjacent forests after the large conflagration of 1988. They concluded that intensive management practices such as the removal of standing and fallen dead material, appeared to reduce the severity of fire damage. Birk and Bridges (1989) conducted a long term experiment on the effects of fuels management and concluded that under a prescribed burning regime, wildfires would be less intense, have lower rates of spread, and therefore could be more easily confined to smaller areas. Given the long-term relationships between fire management actions and ecological and economic outcomes, it may be possible to employ the principles of control theory to enhance decision making.

Control theory is based on four basic assumptions (Lambert 1985). First, there is a relationship between the decision variable, known as the control variable, and future changes in the condition of a resource, known as the state variable. In fire management this is the relationship between fire management activities S and P and their effects on NVC. Second, management decisions are related through time. For example, a decision today to invest in prescribed burning will affect future prescribed burning expenditures depending on the ecological effects, such as hazard reduction and ecosystem restoration--thus making management decisions dynamic. Third, the state of a resource depends on the initial condition of the resource as well as the effect the control variables may have on that resource over time. For wildland fire management, this is particularly important in that areas with relatively high fuel loading and fire hazard may require more management effort to reduce fire hazard and to restore natural conditions. Finally, it can be assumed that the natural system will achieve a steady state. If managers seek to achieve natural mean fire intervals (MFI) of relatively high fire frequencies and low fireline intensities, the targeted MFI will help to determine the length of time over which to evaluate fire management actions.

The general form of the dynamic problem is specified as follows. The state variable NVC defines the state of the resource that is affected by control variables suppression and presuppression. The model, in its general form, is expressed by the objective function [9] and the state equation [10]:

$$\mathrm{Max}_{s,p} \int_{t_0}^{t_1} -f(S(t),P(t),NVC(t),t)e^{-it}dt \qquad [9]$$

$$\mathrm{S.T.} \quad NVC' = g(S(t),NVC(t),t) \qquad [10]$$

$$\mathrm{NVC}(0) = \mathrm{NVC}_0 \boldsymbol{NVC(T)=NVC_T}$$

To minimize the sum of costs plus NVC, subject to both changes in NVC resulting from management activities, and a targeted mean fire interval, we integrate equation [9] over a specified planning period. The solution will yield optimal functions (paths) for management variables S and P, which will generate an optimal path for the state variable NVC. Management will depend on the path of optimal decisions over a specified planning horizon and the net present value of a stream of decisions rather than the sum of decisions made independently of each other over a series of years. Therefore, S and P indicate paths of decision variables made over the planning horizon rather than annual levels of decision variables (Silberberg 1990). To minimize C+NVC, we maximize the negative of equation [9].

To solve the model we formulate a Lagrangean equation [11] using 1 to represent the marginal value of NVC:

$$L = \int_{t_0}^{t_1} -\{f(S,P,NVC,t) + \lambda(t)[NVC' - g(S,P,NVC,t)]\}e^{-IT}dt \qquad [11]$$

and minimize the sum of fire management expenditures plus net value change over the planning period subject to the state equation. The term 1 represents the marginal value of NVC known as the costate variable. Because equation [11] is expressed partially by differential equations, we break the problem into parts and integrate over two distinct periods defined by the endpoint condition T:

$$L = \int_{0}^{T} -[f(S,P,NVC,t) + \lambda g(S,P,NVC,t) + \lambda' NVC]e^{-it}dt$$

$$+ [\lambda(T)NVC(T)e^{-it} - \lambda(0)NVC(0)] \qquad [12]$$

Rewriting the first two terms in equation [12] using the Hamiltonian equation (Lambert 1985) to separate terms through time yields equation [13], which can be solved using the Hamiltonian conditions:

$$L = \int_{0}^{T} -[H(\lambda,S,P,NVC,t) + \lambda' NVC]e^{-it}dt + [\lambda(T)NVC(T)e^{-it}$$

$$-\lambda(0)NVC(0)] \qquad [13]$$

The first-order conditions for maximization are as follows:

$$0 = \frac{\partial H}{\partial S}, \qquad 0 = \frac{\partial H}{\partial P} \qquad [14]$$

$$-\lambda' = \frac{\partial H}{\partial NVC} \qquad [15]$$

More specifically, equation [14] represents the optimality conditions, and equation [15] is the differential equation of the costate variable. The final condition [16] is the differential equation for the state variable and ensures that the constraint expressed by the state equation [10] is true.

$$NVC' = -\frac{\partial H}{\partial \lambda} \qquad [16]$$

After the fire management problem is specified in terms of the relationships between S, P, and NVC, the Hamiltonian conditions can be used to solve for the optimal paths of the control and state variables to generate the optimal program levels (Lambert 1985). Rather than generating annual estimates for fire management activities, the solution to the control problem will provide the optimal paths for fire management activities over a specified rotation.

Discussion

Theoretical frameworks linking short-term activities to long-term effects and objectives are becoming more important as public land management agencies increasingly embrace the tenets of ecosystem management. This paper develops a general extension to the C+NVC methodology to provide a theoretical foundation that directly addresses those tenets. The distinction between annual and long-term fire programming demonstrates that long run cost minimization and efficiency is ultimately dependent upon the damage caused by wildfire as represented by NVC' and the inter-temporal relationships among fire management activities and economic and ecological outcomes. Furthermore, the control theory model embodies the relationship between fire management activities and their resultant effects on the ecosystem over time. In this way, long-term costs and ecological considerations are embedded in the C+NVC model.

The control theory model may also be used to compare the marginal and relative effectiveness of P and S in the long run. A direct relationship between presuppression expenditures and ecological and economic outcomes may lead to enhanced fire management efficiency and reallocation of expenditures away from suppression. To minimize the cost of fire management applications, current fire management policies could be reviewed with stronger emphasis on the relationship between presuppression activities and long-term ecological impacts while more fully incorporating the estimated cost of suppression. For example, budgeting and planning systems such as the USDA Forest Service's NFMAS use the C+NVC framework to solve for the least cost program level (presuppression). The minimum program cost, however, does not accurately reflect the expected cost of fire management for a given time period because the resulting budget does not include scheduled suppression expenditures beyond initial attack. For large fires requiring more suppression than initial attack, funding comes from an unlimited emergency source (United States Congress 1989). The unlimited suppression funding has the effect of increasing total fire management cost for the season beyond the optimal level. A more comprehensive approach will more directly link annual practices with long-term ecological effects and include the cost of suppression funded by the emergency budget. Such an approach will also evaluate the trade-offs between current spending on presuppression versus future spending on suppression.

Application of the control theory approach will require formal specification of the relationships between S, P, and NVC--in particular, the relationship between suppression and presuppression over time. Furthermore, it may be desirable to identify and include a variety of control variables for several fire management activities irrespective of general expenditure category. Finally, in choosing a control variable, it may be beneficial to use a measure other than the traditional net value change. If ecological variables better describe the state or value of a resource, and can be measured, tracking the change in the state variable over time could provide better links between fire management objectives and resulting changes in the ecosystem.

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